# UNIT - 5

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# ASSIGNMENT - 5

## "INTEGRATION"

★ **Properties of double integral :-**

① If K is a constant function then integration are

$$\iint_R K f(x,y)\, dx\, dy = K \iint_R f(x,y)\, dx\, dy$$

② Linear property of double integral

$$\iint_R \{K_1 f(x,y) + K_2 f(x,y)\}\, dx\, dy =$$

$$K_1 \iint_R f_1(x,y)\, dx\, dy + K_2 \iint_R f_2(x,y)\, dx\, dy$$

③ If the region (R) participants into two region $R_1$ & $R_2$ then -

$$\iint_R f(x,y)\, dx\, dy = \iint_{R_1} f(x,y)\, dx\, dy + \iint_{R_2} f(x,y)\, dx\, dy$$

(4) The double integral $\int_a^b \int_{f_1(x)}^{f_2(x)} dx\,dy$ defines the area enclosed by the $y = f_1(x)$, $y = f_2(x)$ & the ordinates $x = a$, $x = b$.

**Que. 1** Evaluate :-

$$\int_0^1 \int_0^{x^2} e^{y/x}\, dy\, dx$$

**Sol.$^n$** ⇒

$$\int_0^1 \left\{ \int_0^{x^2} e^{y/x}\, dy \right\} dx$$

$$\int_0^1 \left[ x\, e^{y/x} \right]_0^{x^2} dx$$

$$\int_0^1 \left[ x\, e^{x^2/x} - x\, e^{0/x} \right] dx$$

$$\int_0^1 (x e^x - x)\, dx$$

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$$\int_0^1 x(e^x - 1)\,dx$$

$$\left[xe^x - e^x - \frac{x^2}{2}\right]_0^1$$

$$\left(1e^1 - e^1 - \frac{1^2}{2}\right) - \left(0e^0 - e^0 - \frac{0^2}{2}\right)$$

$$\left(\cancel{e} - \cancel{e} - \frac{1}{2}\right) - \left(-1 - 0\right)$$

$$\frac{-1}{2} + 1$$

$$\frac{+1}{2}$$ **Ans.**

★ Rough Work –

Let,

$$\int e^{y/x}\,dy \Rightarrow \boxed{y/x = t}$$

$y = xt$

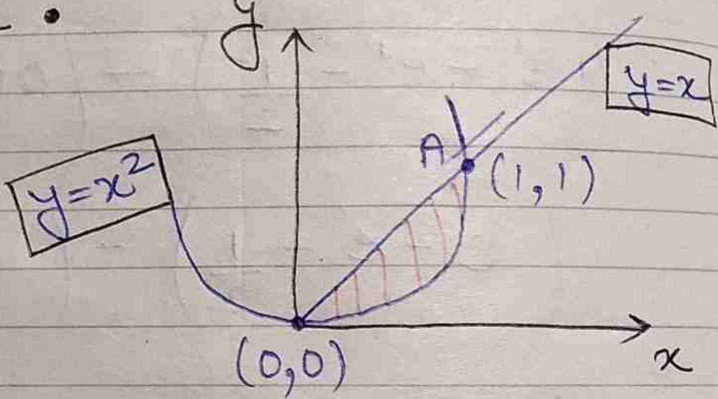
$\boxed{dy = x\,dt}$

$$\int e^t x\,dt = x\int e^t\,dt$$

$$x e^t \Rightarrow xe^{y/x}$$ **Ans.**

**Que.** Evaluate $\iint_R xy(x+y)\,dx\,dy$ over the region (R) bounded by the curve and $y=x^2$ & $y=x$.

**Sol$^n$.** =>




At the point of intersection given curve we have $x=x^2 \Rightarrow x=0,1$ thus the curve intersect at the point $(0,0)$ & $(1,1)$ thus the (R) is the area given by (R):

$$\boxed{0 \le x \le 1} \text{ and } \boxed{x^2 \le y \le x}$$

$$\int_0^1\int_{x^2}^{x} xy\,(x+y)\,dy\,dx$$

$$\int_0^1 \left[\left[x^2\frac{y^2}{2} + x\frac{y^3}{3}\right]_{x^2}^{x}\right] dx$$

$$\int_0^1 \left[x^2\cdot\frac{x^2}{2} + \frac{x\cdot x^3}{3} - x^2\cdot\frac{x^4}{2} - x\cdot\frac{x^6}{3}\right] dx$$

$$\int_0^1 \left[\frac{x^4}{2} + \frac{x^4}{3} - \frac{x^6}{2} - \frac{x^7}{3}\right] dx$$

$$\int_0^1 \left[\frac{3x^4 + 2x^4 - 3x^6 - 2x^7}{6}\right] dx$$

$$\frac{1}{6}\int_0^1 \left[5x^4 - 3x^6 - 2x^7\right] dx$$

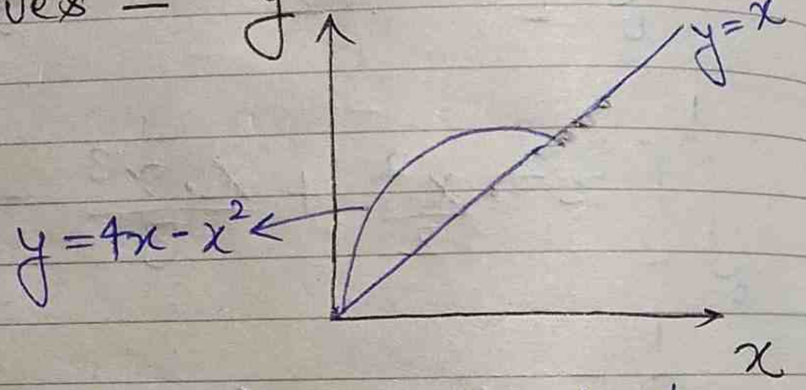
$$\frac{1}{6}\left[\cancel{5}\frac{x^5}{\cancel{5}} - \frac{3x^7}{7} - \frac{2x^8}{8}\right]_0^1$$

$$\frac{1}{6}\left[1^5 - \frac{3(1)^7}{7} - \frac{\cancel{2}(1)^8}{\cancel{8}\,4}\right] \Rightarrow \frac{3}{56}\ \underline{\text{Ans.}}$$

Que. Find the area included b/w the parabola $y = 4x - x^2$ and the line $y=x$

The given region is bonded by the ~~y~~ following Curves –

$$y = 4x - x^2 \quad \text{—(1)}$$
$$y = x \quad \text{—(2)}$$




▸ At the point of intersection of first & second we have, $x = 4x - x^2$

$$x^2 = 4x - x$$
$$x^2 = 3x$$
$$x^2 - 3x = 0$$
$$x(x-3) = 0$$
$$\boxed{x = 0, 3}$$

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Thus, the required area is the shaded area & expressed by $0 \le x \le 3$, $x \le y \le 4x-x^2$

- Hence, the Required Area

$$\int_0^3 \int_x^{4x-x^2} dx\, dy \Rightarrow \int_0^3 \left[\int_x^{4x-x^2} dy\right] dx$$

$$\int_0^3 \left[[y]_x^{4x-x^2}\right] dx$$

$$\int_0^3 [4x-x^2-x]\, dx$$

$$\int_0^3 [3x-x^2]\, dx$$

$$\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3$$

$$\left[\frac{3(3)^2}{2} - \frac{(3)^3}{3}\right] \Rightarrow \frac{27}{2} - \frac{27}{3} \Rightarrow \frac{27\times3 - 27\times2}{6}$$

$$\Rightarrow \frac{81-54}{6} = \frac{\cancel{27}^{9}}{\cancel{6}_{2}} = 4.5 \text{ sq. unit}$$

Ans.

★ EXTRA QUESTIONS –

Que. Evaluate $\int_0^1 \int_0^1 xy \, dx \, dy$

Sol.$^n$ $\Rightarrow$ $\int_0^1 \left[ \int_0^1 xy \, dy \right] dx$

$$\int_0^1 \left[ \frac{xy^2}{2} \right]_0^1 dx$$

$$\int_0^1 \left[ \frac{x(1)^2}{2} \right] dx$$

$$\int_0^1 \left[ \frac{x}{2} \right] dx \Rightarrow \frac{1}{2} \int_0^1 x \, dx$$

$$\frac{1}{2} \left[ \frac{x^2}{2} \right]_0^1 \Rightarrow \frac{1}{2} \left[ \frac{1^2}{2} - \frac{0^2}{2} \right]$$

$\Rightarrow \frac{1}{4}$ Ans.

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**Que.** Evaluate $\int_0^2 \int_0^1 (x^2+y^2)\,dx\,dy$

$$\int_0^2 \left[ \int_0^1 (x^2+y^2)\,dy \right] dx$$

$$\int_0^2 \left[ \left[ x^2 y + \frac{y^3}{3} \right]_0^1 \right] dx$$

$$\int_0^2 \left[ x^2(1) + \frac{(1)^3}{3} \right] dx$$

$$\int_0^2 \left[ x^2 + \frac{1}{3} \right] dx$$

$$\left[ \frac{x^3}{3} + \frac{x}{3} \right]_0^2 \Rightarrow \left[ \frac{x^3+x}{3} \right]_0^2$$

$$\left[ \frac{2^3+2}{3} - \frac{0^3+0}{3} \right] \Rightarrow \left[ \frac{8+2}{3} \right]$$

$$\boxed{= \frac{10}{3}}$$ Ans.

Que. Evaluate $\int_0^4 \int_0^x \int_0^{x+y} z \, dz \, dy \, dx$

Sol.$^n$ ⇒

$$\int_0^4 \int_0^x \left[ \int_0^{x+y} z \, dz \right] dy \, dx$$

$$\int_0^4 \int_0^x \left[ \frac{z^2}{2} \right]_0^{x+y} dy \, dx$$

$$\int_0^4 \int_0^x \left[ \frac{(x+y)^2}{2} \right] dy \, dx$$

$$\int_0^4 \int_0^x \left[ \frac{x^2+y^2+2xy}{2} \right] dy \, dx$$

$$\int_0^4 \frac{1}{2} \left[ \int_0^x [x^2+y^2+2xy] \, dy \right] dx$$

$$\int_0^4 \frac{1}{2} \left[ x^2 y + \frac{y^3}{3} + \frac{\cancel{2}xy^2}{\cancel{2}} \right]_0^x dx$$

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$$\int_0^4 \frac{1}{2}\left[x^3(x) + \frac{x^3}{3} + x.x^2\right] dx$$

$$\frac{1}{2}\int_0^4 \left[x^4 + \frac{x^3}{3} + x^3\right] dx$$

$$\frac{1}{2}\left[\frac{x^5}{5} + \frac{x^4}{3\times 4} + \frac{x^4}{4}\right]_0^4$$

$$\frac{1}{2}\left[\frac{x^5}{5} + \frac{x^4}{12} + \frac{x^4}{4}\right]_0^4$$

$$\frac{1}{2}\left[\frac{(4)^5}{5} + \frac{(4)^4}{12} + \frac{(4)^4}{4}\right]$$

$$\frac{1}{2}\left[\frac{1024}{5} + \frac{\overset{64}{\cancel{256}}}{\underset{3}{\cancel{12}}} + \frac{\overset{64}{\cancel{256}}}{\cancel{4}}\right]$$

$$\frac{1}{2}\left[\frac{1024}{5} + \frac{64}{3} + \frac{64}{1}\right]$$

$$\frac{1}{2}\left[\frac{1024\times 3 + 64\times 5 + 64\times 15}{15}\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{3062+320+960}{15}\right]$$

$$\Rightarrow \frac{1}{\cancel{2}}\left[\frac{\overset{2171}{\cancel{4342}}}{15}\right]$$

$$\Rightarrow \frac{2171}{5}$$ Ans.